

AD-A068 186

CENTER FOR NAVAL ANALYSES ARLINGTON VA  
A SECRETARY PROBLEM WITH A RANDOM NUMBER OF CHOICES.(U)  
MAR 79 K S GLASSER

F/G 12/1

UNCLASSIFIED

CNA-PP-249

NL

| OF |  
AD  
A068186



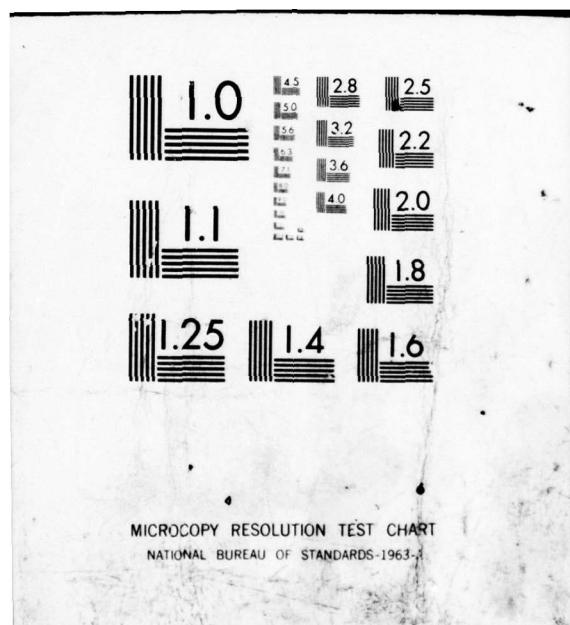
END

DATE

FILMED

7--79

DDC



AD A068186

DDC FILE COPY

② LEVEL

55 000249.00

⑥ A SECRETARY PROBLEM WITH A RANDOM  
NUMBER OF CHOICES.

⑩ Kenneth S. Glasser

⑨ Professional Paper No. 249

⑪ March 1979

⑭ CNA-PP-249

⑫ 32p.

DDC

RECEIVED  
MAY 7 1979  
B

DISTRIBUTION STATEMENT A

Approved for public release  
Distribution Unlimited

The ideas expressed in this paper are those of the  
author. The paper does not necessarily represent  
the views of the Center for Naval Analyses.

CENTER FOR NAVAL ANALYSES

1401 Wilson Boulevard  
Arlington, Virginia 22209

071 270 79 05 02 013

job

A SECRETARY PROBLEM  
WITH A RANDOM NUMBER OF CHOICES

by

Kenneth S. Glasser

Submitted for publication in Journal of the  
American Statistical Association

ABSTRACT

In this paper, the best-choice Secretary Problem is modified to allow the player to make more than one choice. The probability of selecting the best object is computed. The optimal starting time is characterized, and expressions are derived for the expected number of objects chosen and the expected number of objects sampled by this procedure. Asymptotic results are also derived.

KEY WORDS: Secretary Problem, Optimal Stopping Rules, Relative Ranks.

ACCESSION for		
NTIS	White Section <input checked="" type="checkbox"/>	
DDC	Buff Section <input type="checkbox"/>	
UNANNOUNCED <input type="checkbox"/>		
JUSTIFICATION		
<b>PER LETTER</b>		
BY		
DISTRIBUTION/AVAILABILITY CODES		
Dist.	All and/or SPECIAL	
A		

79 05 02 015

\*Kenneth S. Glasser is a Scientific Analyst, Operations Evaluation Group, Center for Naval Analyses, Arlington, Virginia 22209. This work is a part of the author's Ph.D. dissertation, prepared at The American University. The author wishes to thank his advisor, Austin M. Barron, for his help and encouragement.

## 1. INTRODUCTION

In this article, we will consider a procedure that is closely related to the optimal procedure for the best choice Secretary Problem. The problem is formulated as follows:  $N$  objects or individuals can be ranked from best (rank=1) to worst (rank=N). As each object is shown to the experimenter (or player), he is able to rank it only in relation to those objects that he has already seen. The player may select the current object, at which point the procedure ends, or he may elect to reject the current object and sample the next. Once an object has been rejected, it may not be chosen later. If no choice has been made, the  $N^{\text{th}}$  object must be chosen.

In the best choice Secretary Problem, the player tries to maximize the probability of selecting the best object. The optimal procedure, first derived by Lindley (1961), has the following form: Beginning with the  $r^{\text{th}}$  object, choose the first object that is better than all others seen so far. Following Gilbert and Mosteller (1966), we shall call such an object a candidate.  $r$  is called the starting time.

The optimal starting time,  $r^*$ , is given by:

$$r^* = \min \left\{ r \mid \sum_{k=r}^{N-1} (1/k) < 1 \right\} . \quad (1.1)$$

Tables of  $r^*$  and the probability of success for this procedure are given in Gilbert and Mosteller (1966), for  $N=1(1)50(10)100,1000$ .

As  $N$  becomes large, the probability of choosing the best object using the optimal procedure decreases to  $1/e$ , or less than 0.37. It is reasonable to ask if this probability of success can be increased if some of the restrictions placed on the problem are relaxed. The following procedure is proposed to improve the success rate. Note that by using this procedure, it is quite possible for the player to choose more than one object.

Procedure A: Do not select any of the first  $s-1$  objects. Choose the next candidate encountered. Sample the next object; if it is a candidate, select it also. Continue sampling until either a noncandidate or the  $N^{\text{th}}$  object appears. If no objects have been chosen, the  $N^{\text{th}}$  object must be selected.

When the player succeeds in choosing the best object, it will be called a correct selection (CS). The probability of success using Procedure A will be given by  $P_A(\text{CS}; s, N)$ , or by  $P_A(\text{CS})$  when it is clear that  $s$  and  $N$  are given fixed values. The player chooses  $s$  to maximize the probability of having the best object among those selected.

Procedure A is equivalent to the finite memory Secretary Problem (FM) of Smith and Deely (1975) when  $m=2$ , in that the probability of success is the same in both cases. FM gives the player only one choice, however. Also, the definition of starting times are somewhat different; the optimal starting time of FM is always one greater than that of Procedure A. Finally,

FM can be generalized by allowing  $m$  to vary, as Smith and Deely have done. Procedure A will be generalized in Section 5 by allowing the player to encounter several noncandidates before he is forced to stop the procedure.

## 2. COMPUTING THE PROBABILITY OF SUCCESS

We begin by defining several symbols and events. First, the arrival time of the best object will be denoted by  $a^*$ . For example,  $a^*=5$  means that the best item was sampled fifth.  $a_k$  and  $b_k$  will denote the arrival times of the relatively best and second best objects respectively, in the first  $k$  sampled.

The player succeeds when candidates are chosen at times  $k, k+1, \dots, j-1$  and the best object is chosen at time  $j$ . This event will be denoted by  $W(k, j)$ , where  $k \geq s$ .  $W(k, j)$  can occur only if the second best of the first  $k$  items seen has been observed before  $s$ . In terms of the above notation, we can write  $W(k, j)$  as:

$$W(k, j) = [b_k < s, a_k = k, a_{k+1} = k+1, \dots, a_{j-1} = j-1, a^* = j].$$

The probability that  $W(k, j)$  occurs may be computed by first noting:

$$P(W(k, j)) = P(b_k < s, a_k = k, \dots, a_{j-1} = j-1 | a^* = j) P(a^* = j). \quad (2.1)$$

The probability that the best object arrives at any particular time  $j$  is given by  $1/N$ , for  $j=1, \dots, N$ . Now, assume that  $s > 1$ . Given that the best object arrives at time  $j$ , the only restriction on the first  $k-1$  objects is that the best (the second best out of  $k$ ) must arrive before  $s$ . So there are  $k-2$  objects that can be permuted (in  $(k-2)!$  ways) in any order, without affecting the outcome of  $W(k, j)$ . Also, the best object

of the first  $k-1$  may arrive any time before  $s$ , or in any of  $s-1$  times. There are  $(j-1)!$  possible permutations of the first  $j-1$  objects. Combining this in (2.1) yields, for  $k=s, \dots, N$  and  $j=k, \dots, N$ :

$$P(W(k,j)) = (s-1) \cdot f(k-2, j-1)/N , \quad (2.2)$$

$$\text{where } f(k,j) = k!/j! \quad (2.3)$$

When  $s=1$ , the first object is automatically selected, since it is by definition a candidate. The player will win only if every object sampled is a candidate until the best object appears. Thus:

$$P(W(1,k)) = f(0, k-1)/N. \quad (2.4)$$

Summing (2.2) and (2.4) over all possible values of  $j$  and  $k$  yields:

$$P_A(CS; s, N) = \frac{1}{N} \sum_{k=1}^N f(0, k-1) , \quad \text{if } s=1 \quad (2.5)$$

$$= \frac{s-1}{N} \sum_{k=s}^N \sum_{j=k}^N f(k-2, j-1) , \quad \text{if } s>1 .$$

We next calculate the expected number of objects chosen using Procedure A. First, let  $s>1$ .

The player selects objects arriving at  $k, k+1, \dots, j$ , if and only if all of these objects are candidates and the  $(j+1)^{\text{st}}$  object is not a candidate. If this occurs,  $j-k+1$  objects are chosen. For any  $i$ , the probability that the  $i^{\text{th}}$  object is a

candidate is  $1/i$ ,  $i=1, \dots, N$ . The probability that the  $i^{\text{th}}$  object is not a candidate is then  $(i-1)/i$ . Since the objects appear in random order, the relative rank of the  $i^{\text{th}}$  object is independent of the ranks of the first  $i-1$  objects. Thus,

$$P(a_k=k, \dots, a_j=j=a_{j+1}) = j \cdot f(k-1, j+1). \quad (2.6)$$

The arrival time of the best object of the first  $k-1$  is independent of the relative ranks of the objects arriving at times  $k, k+1, \dots, j+1$ , so that:

$$P(b_k < s | a_k=k, \dots, a_j=j=a_{j+1}) = \frac{s-1}{k-1}. \quad (2.7)$$

Finally, the probability of selecting  $j-k+1$  objects is given by multiplying (2.6) and (2.7):

$$P(b_k < s, a_k=k, \dots, a_j=j=a_{j+1}) = (s-1) \cdot f(k-2, j-1) / (j+1), \quad (2.8)$$

for  $j = k, k+1, \dots, N-1$ , and  $k = s, \dots, N-1$ .

The probability of selecting the best object at time  $N$ , when the first choice was made of a candidate arriving at time  $k$ , is given by:

$$P(b_k < s, a_k=k, \dots, a_{N-1}=N-1, a^*=N) = (s-1) \cdot f(k-2, N). \quad (2.9)$$

Equation (2.9) describes an event where  $N-k+1$  objects are chosen.

If no candidates are encountered after time  $s-1$ , the player must choose the  $N^{\text{th}}$  object. He will make one choice of

a noncandidate at time  $N$  with probability:

$$P(a^* < s) = (s-1)/N. \quad (2.10)$$

Denote the number of objects chosen by the random variable  $X$ . The expected value of  $X$  is given by multiplying (2.8), (2.9), and (2.10) by the number of objects chosen and then summing over their respective values of  $j$  and  $k$ :

$$E_A(X; s, N) = (s-1) \sum_{k=s}^{N-1} \sum_{j=k}^{N-1} \frac{j-k+1}{j+1} f(k-2, j-1)$$

$$+ (s-1) \sum_{k=s}^N (N-k+1) f(k-2, N) + \frac{s-1}{N}, \quad (2.11)$$

for  $s > 1$ .

For  $s=1$ , the equations are simpler in form. We can compute:

$$E_A(X; 1, N) = \sum_{k=1}^N k^2 \cdot f(0, k+1) + f(0, N-1). \quad (2.12)$$

The  $m^{\text{th}}$  moment can be calculated in the same manner as (2.11) and (2.12).

We can also calculate the expected number of objects sampled before the procedure ends. First, for  $s > 1$ , the probability that  $j+1$  objects are sampled when the first choice is made at  $k \geq s$  is given by (2.8). The probability that  $N$  objects are sampled when the first choice is made at  $k$  is given by (2.9). The probability that  $N$  objects are sampled, and the

only choice is made of a noncandidate at  $N$  is given by (2.10).

Denote the number of objects sampled by the random

variable  $Y$ . Then, the expected value of  $Y$  is given by:

$$\begin{aligned} E_A(Y; s, N) &= \sum_{k=s}^{N-1} \sum_{j=k}^{N-1} (j+1) \frac{s-1}{j+1} f(k-2, j-1) \\ &\quad + \sum_{k=s}^N (s-1)N'f(k-2, N) + N \cdot \frac{s-1}{N} \\ &= N P_A(CS; s, N) + (s-1). \end{aligned} \tag{2.13}$$

It can be shown that (2.13) also holds for  $s=1$ .

### 3. FINDING THE OPTIMUM S

Equation (2.5) gives the probability of success using Procedure A. The following lemma shows the existence of a starting point  $s^*$  that maximizes  $P_A(CS)$  for a given  $N$ .

Lemma 3.1: There exists an  $s$ ,  $1 \leq s \leq N$ , that maximizes  $P_A(CS)$  for a fixed  $N$ . For any  $s' < s$ ,  $P_A(CS; s', N) \leq P_A(CS; s, N)$ .

For any  $s' > s$ ,  $P_A(CS; s, N) \geq P_A(CS; s', N)$ .

Proof: For any  $s$ ,

$$\begin{aligned} F(s; N) &= N \left[ P_A(CS; s, N) - P_A(CS; s+1, N) \right] \\ &= 1 - \sum_{k=s+2}^N \sum_{j=k}^N f(k-2, j-1). \end{aligned} \tag{3.1}$$

$F(s; N)$  is a non-decreasing function of  $s$ , since:

$$\begin{aligned} F(s+1; N) - F(s; N) &= \sum_{j=s+2}^N f(s, j-1) \\ &\geq 0. \end{aligned}$$

Since  $F(s; N)$  is non-decreasing in  $s$ , if  $F(s; N) \geq 0$ , then  $F(s+1; N) \geq 0$ . This completes the proof.

The optimum  $s$  for a given  $N$  is found by using (3.1), since  $P_A(CS; s, N) > P_A(CS; s+1, N)$  if  $F(s; N) > 0$ . Thus, the  $s$  that maximizes  $P_A(CS; s, N)$  is given by:

$$s^* = \min \left[ s \mid 1 > \sum_{k=s+2}^N \sum_{j=k}^N f(k-2, j-1) \right]. \quad (3.2)$$

In practice, (3.2) does not substantially reduce the amount of work necessary to find  $s^*$ . The double sum in (3.2) can be computed using backwards iteration, since:

$$\sum_{k=s}^N \sum_{j=k}^N f(k-2, j-1) = \sum_{j=s}^N f(s-2, j-1) + \sum_{k=s+1}^N \sum_{j=k}^N f(k-2, j-1). \quad (3.3)$$

When  $F(s; N) = 0$ , from (3.1) it can be seen that  $P_A(CS; s, N) = P_A(CS; s+1, N)$ . In this case,  $s^*$  could be taken to be either  $s$  or  $s+1$ . We assume that the player will wish to take the  $s^*$  that gives the lower value of  $E_A(X; s, N)$ .

Table 1 contains values of  $s^*$ ,  $P_A(CS; s^*, N)$ ,  $E_A(X; s^*, N)$ , and  $E_A(Y; s^*, N)$  for  $N = 1(1) 50(10) 100, 1000$ . For small  $N$ , the probability of success is quite a bit better than that of the optimal procedure of the best-choice Secretary Problem. As  $N$  becomes large, however, there is not much difference. This is because as  $s^*$  increases, the probability of making more than one choice decreases. The next section will cover asymptotic results in detail.

Also, as mentioned in section 2.1, some of the results can be compared with certain results obtained by Smith and Deely (1975). In their table (p.361), for  $m=2$  and  $N=10, 50, \text{and } 100$ , the probabilities of success agree exactly. Note that their optimal starting time  $r^*$  is always one greater than the optimal starting time  $s^*$  derived in this section. This is due to a difference in the definitions of  $r^*$  and  $s^*$ .

#### 4. ASYMPTOTIC RESULTS

In this section, we will study  $P_A(CS)$ ,  $E_A(X)$ , and  $E_A(Y)$  as  $N$  increases to infinity. First, let  $s^*(N)$  be the optimal starting time when picking the best of  $N$  objects using Procedure A. When there is no ambiguity,  $s^*(N)$  will be written as  $s^*$ .

Lemma 4.1  $s^*(N+1) \geq s^*(N)$  .

Proof: For any  $r < s^*(N)$ :

$$\sum_{k=r+2}^{N+1} \sum_{j=k}^{N+1} f(k-2, j-1) > \sum_{k=r+2}^N \sum_{j=k}^N + (k-2, j-1) \\ > 1 .$$

The last inequality follows from the definition of  $s^*(N)$  in (3.2) and since  $r < s^*(N)$  .

Lemma 4.2  $\lim_{N \rightarrow \infty} s^*(N) = \infty$  .

Proof: We will prove this lemma by showing that there exists an  $N_2 > N_1$  large enough so that  $s^*(N_2) > s^*(N_1)$ . Note first, that for any  $s$  :

$$\sum_{k=s+2}^N \sum_{j=k}^N f(k-2, j-1) > \sum_{k=s+2}^N 1/(k-1) . \quad (4.1)$$

As  $N$  increases to infinity, the right hand side of (4.1) diverges. For  $N=N_1$  and  $s=s^*(N_1)$ , the left hand side of (4.1) must be less than one, by (3.2), but there are an infinite number of  $N_2$ 's large enough to make the entire inequality greater than one. Define:

$$N_2 = \min \left[ M \mid \sum_{k=s^*(N_1)+2}^M \frac{1}{(k-1)} > 1 \right].$$

Then  $s^*(N_2) > s^*(N_1)$ .

Corollary 4.1  $\lim_{N \rightarrow \infty} E_A(Y) = \infty$ .

Proof: This result follows directly from Lemma 4.2, since the player must wait at least until  $s^*(N)$  to make his first choice.

Theorem 4.1:  $\lim_{N \rightarrow \infty} (s^*/N) = \lim_{N \rightarrow \infty} P_A(CS; s^*, N) = 1/e$ . (4.2)

Proof: It is well known (Lindley (1961), etc.) that (4.2) holds for the optimal procedure for the best-choice Secretary Problem. We will prove this theorem by showing that the optimal procedure and Procedure A are asymptotically equivalent. Thus, (4.2) holds for Procedure A also.

For  $s > 1$ , the probability that one object is chosen at  $k$  is given by:

$$P[b_k < s, a_k = k = a_{k+1}] = \frac{s-1}{(k-1)(k+1)}, \quad k=s, \dots, N-1.$$

One candidate is chosen at time  $N$  with probability:

$$P(b_N < s, a^* = N) = \frac{s-1}{(N-1)N} .$$

Finally, one object is chosen if the player is forced to choose a non-candidate at time  $N$ . The probability that this occurs is given by equation (2.10).

Thus,

$$\begin{aligned} P(X=1; s, N) &= (s-1) \left[ \sum_{k=s}^{N-1} \frac{1}{(k-1)(k+1)} + \frac{1}{(N-1)N} + \frac{1}{N} \right] \\ &= 1 - \frac{1}{2s} + \frac{1}{2(N-1)} \frac{s}{N} - \frac{1}{2N(N-1)} . \end{aligned} \quad (4.3)$$

Take the value of  $s$  to be the optimum starting time  $s^*$  in (4.3). By allowing  $N$  to increase to infinity, we have:

$$\lim_{N \rightarrow \infty} P(X=1; s^*, N) = 1 . \quad (4.4)$$

This proves the theorem.

$P(X=1; s^*, N)$  approaches unity quite rapidly. Even for  $N$  as low as 12, the probability that exactly one object will be chosen exceeds 0.90.

Corollary 4.2:  $\lim_{N \rightarrow \infty} E_A(X; s^*, N) = 1.$

Proof: The corollary is a direct result of (4.4).

Corollary 4.3:  $\lim_{N \rightarrow \infty} [E_A(Y; s^*, N)/N] = 2/e .$

Proof: The result follows directly from (2.13) and Theorem 4.1.

## 5. SAMPLING UNTIL TWO (OR MORE) NON-CANDIDATES ARE ENCOUNTERED

One way to increase the probability of selecting the best object is to use the following procedure:

Procedure  $A_u$ : Select an integer  $u \leq N$ . Sample  $s-1$  objects without selecting any. Beginning with the  $s^{\text{th}}$  object, choose the first candidate that appears. Continue to sample objects, selecting candidates and rejecting non-candidates. The procedure is stopped if either the number of non-candidates sampled after the first choice was made is equal to  $u$  or the  $N^{\text{th}}$  candidate is sampled. If no choices have been made, the  $N^{\text{th}}$  object must be selected.

Denote the probability of success using  $A_u$  by  $P_{A_u}(\text{CS}; s, N)$  or by  $P_{A_u}(\text{CS})$  when it is clear that  $s$  and  $N$  are given fixed values. The player is interested in choosing  $s$  to maximize  $P_{A_u}(\text{CS})$ .

Using Procedure  $A_u$ , the player selects the best object in one of two mutually exclusive ways:

(1) The first candidate is chosen at some time  $k \geq s$ .

Up to  $u-2$  non-candidates appear between times  $k$  and  $j$ .

At time  $j$ , the best object is chosen. All of the succeeding objects are non-candidates with probability one.

(2) Exactly  $u-1$  non-candidates appear between times  $k$  and  $j$ . The  $(j+1)^{st}$  object is a non-candidate with probability one.

The probability that event (1) of the preceding paragraph occurs is given by  $P_{A_{u-1}}(CS)$ . If we then denote event (2) by  $W_{u-1}(s, k, j)$ , we have:

$$P_{A_u}(CS; s, N) = P_{A_{u-1}}(CS; s, N) + P(W_{u-1}(s)) . \quad (5.1)$$

To compute the last term in (5.1), first assume that  $u-1$  non-candidates appear at times  $m_1, m_2, \dots, m_{u-1}$ , and that  $s > 1$ . Then,

$$P(W_{u-1}(s, k, j, m_1, \dots, m_{u-1})) = \frac{s-1}{N} f(k-2, j-1) \prod_{i=1}^{u-1} (m_i - 1) , \quad (5.2)$$

where  $s \leq k \leq N-u$ ,  $k+u \leq j \leq N$ , and  $k < m_1 < m_2 < \dots < m_{u-1} < j$ .

Next, sum (5.2) on  $m_1, m_2, \dots, m_{u-1}$ :

$$P(W_{u-1}(s, k, j)) = \frac{s-1}{N} f(k-2, j-1) F(k, j) , \quad (5.3)$$

where

$$F(k, j) = \sum_{m_1=k+1}^{j-u+1} \sum_{m_2=m_1+1}^{j-u+2} \dots \sum_{m_{u-1}=m_{u-2}+1}^{j-1} \prod_{i=1}^{u-1} (m_i - 1) .$$

Summing (5.3) over  $k$  and  $j$  yields, for  $s > 1$ :

$$P(W_{u-1}(s)) = \frac{s-1}{N} \sum_{k=s}^{N-u} \sum_{j=k+u}^N f(k-2, j-1) F(k, j) . \quad (5.4)$$

When  $s=1$ , the calculations are similar:

$$P(W_{u-1}(1)) = \frac{1}{N} \sum_{j=u+1}^N f(l, j-1) F(l, j) . \quad (5.5)$$

Equation (5.1) can then be evaluated by forward iteration, begining with equation (2.5) and then using equation (5.4) or (5.5).

While (5.1) does give an expression by which  $P_{A_u}(CS)$  can be calculated, it is not of much practical use. For  $u>2$ , it is too time consuming to compute the equations by hand. It is possible to program (5.4) and (5.5), but this would also be difficult to do except for the smallest  $N$ , given the usual program size and time limitations at most installations.

When  $u=2$ , however, (5.1) can be put into a usable form, since  $W_1(s)$  can be written as a function of  $P_A(CS)$ . First, for  $s>1$ , (5.3) takes the form:

$$P(W_1(s, k, j)) = \frac{s-1}{2N} [f(k-2, j-3) - f(k, j-1)] . \quad (5.6)$$

Then, by summing (5.6) over  $j$  and  $k$  we have:

$$P(W_1(s)) = \frac{1}{2} \frac{N-2}{N} P_A(CS; s, N-2) - \frac{1}{2} \frac{s-1}{s+1} P_A(CS; s+1, N) . \quad (5.7)$$

When  $s=1$ , (5.5) becomes:

$$P(W_1(1)) = \frac{1}{2} \frac{N-2}{N} P_A(CS; 1, N-2) . \quad (5.8)$$

Thus, for  $u=2$ , (5.1) can be written as:

$$\begin{aligned} P_{A_2}(CS; s, N) &= P_A(CS; s, N) + \frac{1}{2} \frac{N-2}{N} P_A(CS; s, N-2) \\ &\quad - \frac{1}{2} \frac{s-1}{s+1} P_A(CS; s+2, N) . \end{aligned} \tag{5.9}$$

Table 2 contains the optimum starting time  $s^*$  and the probability of correct selection  $P_{A_2}(CS; s^*, N)$  for  $N=1(1)50(10)100,1000$ . The values of  $s^*$  were found by calculating  $P_{A_2}(CS; s, N)$  for all  $s$  for a given  $N$ . As in section 3, the optimum value of  $s$  may be characterized.  $s^*$  is the smallest  $s$  such that:

$$\begin{aligned} 2 > \sum_{k=s+2}^N \sum_{j=k}^N f(k-2, j-1) + \frac{1}{2} \sum_{k=s+2}^{N-2} \sum_{j=k}^{N-2} f(k-2, j-1) \\ &\quad + \frac{1}{2} \frac{s-1}{s+1} - \sum_{j=s+3}^N f(s, j-1) - \frac{1}{2} \sum_{k=s+4}^N \sum_{j=k}^N f(k-2, j-1) . \end{aligned} \tag{5.10}$$

Equation (5.10) is not of much practical use.

The optimum  $s$  for  $P_{A_2}(CS)$  heuristically appear to be equal to or at most one less than the optimum  $s$  for  $P_A(CS)$  for a given  $N$ . Note that for  $N=1000$ ,  $P_{A_2}(CS; 368, 1000) = .369462$  while  $P_A(CS; 368, 1000) = .368829$ . Thus, the player gains only a 0.17 percent advantage by using  $A_2$  instead of  $A$ . Under the assumption that  $s^*$  increases without bound, as seems to be the case, it can be shown in a manner similar to theorem 4.1 that  $A_2$

is asymptotically equivalent to the optimal procedure of the best choice Secretary Problem.

1. Optimal Starting Time, Probability of Success, Expected Number  
Selected and Expected Sample Size For Procedure A

N	s*	P <sub>A</sub> (CS)	E <sub>A</sub> (X)	E <sub>A</sub> (Y)	N	s*	P <sub>A</sub> (CS)	E <sub>A</sub> (X)	E <sub>A</sub> (Y)
1	1	1.0	1.0	1.0	29	11	.402284	1.042	21.67
2	1	1.0	1.5	2.0	30	11	.400795	1.042	22.02
3	1	.833333	1.667	2.500	31	12	.399903	1.038	23.40
4	2	.666667	1.250	3.667	32	12	.398877	1.038	23.76
5	2	.616667	1.283	4.083	33	13	.397655	1.035	25.12
6	2	.561111	1.297	4.367	34	13	.397000	1.035	25.50
7	3	.532143	1.171	5.725	35	13	.396056	1.035	25.86
8	3	.508929	1.179	6.071	36	14	.395189	1.032	27.23
9	4	.488029	1.121	7.392	37	14	.394555	1.032	27.60
10	4	.477447	1.126	7.774	38	14	.393682	1.033	27.96
11	4	.464790	1.129	8.113	39	15	.393078	1.030	29.33
12	5	.456275	1.096	9.475	40	15	.392468	1.030	29.70
13	5	.449412	1.099	9.842	41	15	.391657	1.030	30.06
14	5	.441306	1.101	10.18	42	16	.391252	1.028	31.43
15	6	.437435	1.080	11.56	43	16	.390667	1.028	31.80
16	6	.432542	1.081	11.92	44	17	.390051	1.026	33.16
17	7	.427892	1.066	13.27	45	17	.389657	1.026	33.54
18	7	.425041	1.068	13.65	46	17	.389095	1.027	33.90
19	7	.421316	1.069	14.01	47	18	.388646	1.025	35.27
20	8	.418630	1.058	15.37	48	18	.388252	1.025	35.64
21	8	.416291	1.059	15.74	49	18	.387713	1.025	36.00
22	8	.413320	1.060	16.09	50	19	.387395	1.024	37.37
23	9	.411764	1.051	17.47	60	22	.383985	1.020	44.04
24	9	.409792	1.052	17.84	70	26	.381708	1.017	51.72
25	10	.407709	1.046	19.19	80	30	.379950	1.015	59.40
26	10	.406477	1.046	19.57	90	34	.378554	1.013	67.07
27	10	.404778	1.047	19.93	100	37	.377497	1.012	73.75
28	11	.403405	1.041	21.30	1000	368	.368829	1.001	735.80

2. Optimal Starting Time and Probability of  
Success Using Procedure A<sub>2</sub>

N	s*	P <sub>A<sub>2</sub></sub> (CS)	N	s*	P <sub>A<sub>2</sub></sub> (CS)
1	1	1.0	29	11	.425286
2	1	1.0	30	11	.423490
3	1	1.0	31	11	.421380
4	1	.916667	32	12	.419751
5	1	.791667	33	12	.418168
6	2	.705556	34	12	.416328
7	2	.650000	35	13	.415159
8	3	.605060	36	13	.413746
9	3	.577745	37	14	.412348
10	3	.550033	38	14	.411287
11	4	.533805	39	14	.410015
12	4	.517892	40	15	.408956
13	5	.504109	41	15	.407980
14	5	.494550	42	15	.406825
15	5	.484139	43	16	.406027
16	6	.476812	44	16	.405124
17	6	.470002	45	17	.404159
18	7	.462872	46	17	.403471
19	7	.458410	47	17	.402631
20	7	.453265	48	18	.401874
21	8	.448759	49	18	.401221
22	8	.445204	50	18	.400438
23	8	.441144	60	22	.394941
24	9	.438201	70	26	.390975
25	9	.435280	80	29	.387992
26	10	.432080	90	33	.385756
27	10	.430016	100	37	.383939
28	10	.427558	1000	368	.369462

REFERENCES

Gilbert, John P., and Mosteller, Frederick (1966), "Recognizing the Maximum of a Sequence," Journal of the Americal Statistical Association, 61, 35-73.

Lindley, D.V. (1961), "Dynamic Programming and Decision Theory," Applied Statistics, 10, 39-51.

Smith, M.H., and Deely, J.J. (1975), "A Secretary Problem With Finite Memory," Journal of the American Statistical Association, 70, 357-361.

## CNA Professional Papers – 1973 to Present\*

- PP 103**  
Friedheim, Robert L., "Political Aspects of Ocean Ecology" 48 pp., Feb 1973, published in Who Protects the Oceans, John Lawrence Hargrove (ed.) (St. Paul: West Publ. Co., 1974), published by the American Society of International Law AD 757 936
- PP 104**  
Schick, Jack M., "A Review of James Cable, Gunboat Diplomacy Political Applications of Limited Naval Forces," 5 pp., Feb 1973, (Reviewed in the American Political Science Review, Vol. LXVI, Dec 1972)
- PP 105**  
Corn, Robert J. and Phillips, Gary R., "On Optimal Correction of Gunfire Errors," 22 pp., Mar 1973, AD 761 674
- PP 106**  
Stoloff, Peter H., "User's Guide for Generalized Factor Analysis Program (FACTAN)," 35 pp., Feb 1973, (Includes an addendum published Aug 1974) AD 758 824
- PP 107**  
Stoloff, Peter H., "Relating Factor Analytically Derived Measures to Exogenous Variables," 17 pp., Mar 1973, AD 758 820
- PP 108**  
McConnell, James M. and Kelly, Anne M., "Super-power Naval Diplomacy in the Indo-Pakistani Crisis," 14 pp., 5 Feb 1973, (Published, with revisions, in Survival, Nov/Dec 1973) AD 761 675
- PP 109**  
Berghofer, Fred G., "Salaries—A Framework for the Study of Trend," 8 pp., Dec 1973, (Published in Review of Income and Wealth, Series 18, No. 4, Dec 1972)
- PP 110**  
Augusta, Joseph, "A Critique of Cost Analysis," 9 pp., Jul 1973, AD 766 376
- PP 111**  
Herrick, Robert W., "The USSR's 'Blue Belt of Defense' Concept: A Unified Military Plan for Defense Against Seaborne Nuclear Attack by Strike Carriers and Polaris/Poseidon SSBNs," 18 pp., May 1973, AD 766 375
- PP 112**  
Ginsberg, Lawrence H., "ELF Atmosphere Noise Level Statistics for Project SANGUINE," 29 pp., Apr 1974, AD 786 969
- PP 113**  
Ginsberg, Lawrence H., "Propagation Anomalies During Project SANGUINE Experiments," 5 pp., Apr 1974, AD 786 968
- PP 114**  
Maloney, Arthur P., "Job Satisfaction and Job Turnover," 41 pp., Jul 1973, AD 768 410
- PP 115**  
Silverman, Lester P., "The Determinants of Emergency and Elective Admissions to Hospitals," 145 pp., 18 Jul 1973, AD 766 377
- PP 116**  
Rehm, Allan S., "An Assessment of Military Operations Research in the USSR," 19 pp., Sep 1973, (Reprinted from Proceedings, 30th Military Operations Research Symposium (U), Secret Dec 1972) AD 770 116
- PP 117**  
McWhite, Peter B. and Ratliff, H. Donald,\* "Defending a Logistics System Under Mining Attack,"\*\* 24 pp., Aug 1976 (to be submitted for publication in Naval Research Logistics Quarterly), presented at 44th National Meeting, Operations Research Society of America, November 1973, AD A030 454  
\*University of Florida.  
\*\*Research supported in part under Office of Naval Research Contract N00014-68-0273-0017
- PP 118**  
Barfoot, C. Bernard, "Markov Duels," 18 pp., Apr 1973, (Reprinted from Operations Research, Vol. 22, No. 2, Mar-Apr 1974)
- PP 119**  
Stoloff, Peter and Lockman, Robert F., "Development of Navy Human Relations Questionnaire," 2 pp., May 1974, (Published in American Psychological Association Proceedings, 81st Annual Convention, 1973) AD 779 240
- PP 120**  
Smith, Michael W. and Schrimper, Ronald A., "Economic Analysis of the Intracity Dispersion of Criminal Activity," 30 pp., Jun 1974, (Presented at the Econometric Society Meetings, 30 Dec 1973) AD 780 538  
\*Economics, North Carolina State University.
- PP 121**  
Devine, Eugene J., "Procurement and Retention of Navy Physicians," 21 pp., Jun 1974, (Presented at the 49th Annual Conference, Western Economic Association, Las Vegas, Nev., 10 Jun 1974) AD 780 539
- PP 122**  
Kelly, Anne M., "The Soviet Naval Presence During the Iraq-Kuwaiti Border Dispute: March-April 1973," 34 pp., Jun 1974, (Published in Soviet Naval Policy, ed. Michael McCgwire; New York: Praeger) AD 780 592
- PP 123**  
Petersen, Charles C., "The Soviet Port-Clearing Operation in Bangladesh, March 1972-December 1973," 35 pp., Jun 1974, (Published in Michael McCgwire, et al. (eds) Soviet Naval Policy: Objectives and Constraints, (New York: Praeger Publishers, 1974) AD 780 540
- PP 124**  
Friedheim, Robert L. and John, Mary E., "Anticipating Soviet Behavior at the Third U.N. Law of the Sea Conference: USSR Positions and Dilemmas," 37 pp., 10 Apr 1974, (Published in Soviet Naval Policy, ed. Michael McCgwire; New York: Praeger) AD 783 701
- PP 125**  
Weinland, Robert G., "Soviet Naval Operations—Ten Years of Change," 17 pp., Aug 1974, (Published in Soviet Naval Policy, ed. Michael McCgwire; New York: Praeger) AD 783 962
- PP 126** — Classified.
- PP 127**  
Dragovich, George S., "The Soviet Union's Quest for Access to Naval Facilities in Egypt Prior to the June War of 1967," 64 pp., Jul 1974, AD 786 318
- PP 128**  
Stoloff, Peter and Lockman, Robert F., "Evaluation of Naval Officer Performance," 11 pp., (Presented at the 82nd Annual Convention of the American Psychological Association, 1974) Aug 1974, AD 784 012
- PP 129**  
Holen, Arlene and Horowitz, Stanley, "Partial Unemployment Insurance Benefits and the Extent of Partial Unemployment," 4 pp., Aug 1974, (Published in the Journal of Human Resources, Vol. IX, No. 3, Summer 1974) AD 784 010
- PP 130**  
Dismukes, Bradford, "Roles and Missions of Soviet Naval General Purpose Forces in Wartime: Pro-SBNN Operation," 20 pp., Aug 1974, AD 786 320
- PP 131**  
Weinland, Robert G., "Analysis of Gorshkov's Navies in War and Peace," 45 pp., Aug 1974, (Published in Soviet Naval Policy, ed. Michael McCgwire; New York: Praeger) AD 786 319
- PP 132**  
Kleinman, Samuel D., "Racial Differences in Hours Worked in the Market: A Preliminary Report," 77 pp., Feb 1975, (Paper read on 26 Oct 1974 at Eastern Economic Association Convention in Albany, N.Y.) AD A 005 517
- PP 133**  
Squires, Michael L., "A Stochastic Model of Regime Change in Latin America," 42 pp., Feb 1975, AD A 007 912
- PP 134**  
Root, R. M. and Cunniff, P. F.,\* "A Study of the Shock Spectrum of a Two-Degrees-of-Freedom Non-linear Vibratory System," 35 pp., Dec 1975, (Published in the condensed version of The Journal of the Acoustic Society, Vol 60, No. 6, Dec 1976, pp. 1314  
\*Department of Mechanical Engineering, University of Maryland.
- PP 135**  
Goudreau, Kenneth A.; Kuzmack, Richard A.; Wiedemann, Karen, "Analysis of Closure Alternatives for Naval Stations and Naval Air Stations," 47 pp., 3 Jun 1975 (Reprinted from "Hearing before the Subcommittee on Military Construction of the Committee on Armed Service," U.S. Senate, 93rd Congress, 1st Session, Part 2, 22 Jun 1973)
- PP 136**  
Stallings, William, "Cybernetics and Behavior Therapy," 13 pp., Jun 1975
- PP 137**  
Petersen, Charles C., "The Soviet Union and the Reopening of the Suez Canal: Mineclearing Operations in the Gulf of Suez," 30 pp., Aug 1975, AD A 015 376

\*CNA Professional Papers with an AD number may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22151. Other papers are available from the author at the Center for Naval Analyses, 1401 Wilson Boulevard, Arlington, Virginia 22209.

- PP 138  
Stallings, William, "BRIDGE: An Interactive Dialogue-Generation Facility," 5 pp., Aug 1975 (Reprinted from IEEE Transactions on Systems, Man, and Cybernetics, Vol. 5, No. 3, May 1975)
- PP 139  
Morgan, William F., Jr., "Beyond Folklore and Fables in Forestry to Positive Economics," 14 pp., (Presented at Southern Economic Association Meetings November, 1974) Aug 1975, AD A 015 293
- PP 140  
Mahoney, Robert and Druckman, Daniel\*, "Simulation, Experimentation, and Context," 36 pp., 1 Sep 1975, (Published in Simulation & Games, Vol. 6, No. 3, Sep 1975)  
\*Mathematics, Inc.
- PP 141  
Mizrahi, Maurice M., "Generalized Hermite Polynomials,"\* 5 pp., Feb 1976 (Reprinted from the Journal of Computational and Applied Mathematics, Vol. 1, No. 4 (1976), 273-277).  
\*Research supported by the National Science Foundation
- PP 142  
Lockman, Robert F., Jahn, Christopher, and Shughart, William F. II, "Models for Estimating Premature Losses and Recruiting District Performance," 36 pp., Dec 1975 (Presented at the RAND Conference on Defense Manpower, Feb 1976; to be published in the conference proceedings) AD A 020 443
- PP 143  
Horowitz, Stanley and Sherman, Allan (LCdr, USN), "Maintenance Personnel Effectiveness in the Navy," 33 pp., Jan 1976 (Presented at the RAND Conference on Defense Manpower, Feb 1976; to be published in the conference proceedings) AD A 021 581
- PP 144  
Durch, William J., "The Navy of the Republic of China - History, Problems, and Prospects," 66 pp., Aug 1976 (To be published in "A Guide to Asiatic Fleets," ed. by Barry M. Blechman; Naval Institute Press) AD A030 460
- PP 145  
Kelly, Anne M., "Port Visits and the "Internationalist Mission" of the Soviet Navy," 36 pp., Apr 1976 AD A023 436
- PP 146  
Palmour, Vernon E., "Alternatives for Increasing Access to Scientific Journals," 6 pp., Apr 1975 (Presented at the 1975 IEEE Conference on Scientific Journals, Cherry Hill, N.C., Apr 28-30; published in IEEE Transactions on Professional Communication, Vol. PC-18, No. 3, Sep 1975) AD A021 798
- PP 147  
Kessler, J. Christian, "Legal Issues in Protecting Offshore Structures," 33 pp., Jun 1976 (Prepared under task order N00014-68-A-0091-0023 for ONR) AD A028 389
- PP 148  
McConnell, James M., "Military-Political Tasks of the Soviet Navy in War and Peace," 62 pp., Dec 1975 (Published in Soviet Oceans Development Study of Senate Commerce Committee October 1976) AD A022 590
- PP 149  
Squires, Michael L., "Counterforce Effectiveness: A Comparison of the Tsipis "K" Measure and a Computer Simulation," 24 pp., Mar 1976 (Presented at the International Study Association Meetings, 27 Feb 1976) AD A022 591
- PP 150  
Kelly, Anne M. and Petersen, Charles, "Recent Changes in Soviet Naval Policy: Prospects for Arms Limitations in the Mediterranean and Indian Ocean," 28 pp., Apr 1976, AD A 023 723
- PP 151  
Horowitz, Stanley A., "The Economic Consequences of Political Philosophy," 8 pp., Apr 1976 (Reprinted from Economic Inquiry, Vol. XIV, No. 1, Mar 1976)
- PP 152  
Mizrahi, Maurice M., "On Path Integral Solutions of the Schrodinger Equation, Without Limiting Procedure,"\* 10 pp., Apr 1976 (Reprinted from Journal of Mathematical Physics, Vol. 17, No. 4 (Apr 1976), 566-575).  
\*Research supported by the National Science Foundation
- PP 153  
Mizrahi, Maurice M., "WKB Expansions by Path Integrals, With Applications to the Anharmonic Oscillator,"\* 137 pp., May 1976, AD A025 440  
\*Research supported by the National Science Foundation
- PP 154  
Mizrahi, Maurice M., "On the Semi-Classical Expansion in Quantum Mechanics for Arbitrary Hamiltonians," 19 pp., May 1976 (Published in Journal of Mathematical Physics, Vol. 18, No. 4, p. 786, Apr 1977), AD A025 441
- PP 155  
Squires, Michael L., "Soviet Foreign Policy and Third World Nations," 26 pp., Jun 1976 (Prepared for presentation at the Midwest Political Science Association meetings, Apr 30, 1976) AD A028 388
- PP 156  
Stallings, William, "Approaches to Chinese Character Recognition," 12 pp., Jun 1976 (Reprinted from Pattern Recognition (Pergamon Press), Vol. 8, pp. 87-98, 1976) AD A028 692
- PP 157  
Morgan, William F., "Unemployment and the Pentagon Budget: Is There Anything in the Empty Pork Barrel?" 20 pp., Aug 1976 AD A030 455
- PP 158  
Haskell, LCdr. Richard D. (USN), "Experimental Validation of Probability Predictions," 25 pp., Aug 1976 (Presented at the Military Operations Research Society Meeting, Fall 1976) AD A030 458
- PP 159  
McConnell, James M., "The Gorshkov Articles, The New Gorshkov Book and Their Relation to Policy," 93 pp., Jul 1976 (Published in Soviet Naval Influence: Domestic and Foreign Dimensions, ed. by M. McGwire and J. McDonnell; New York: Praeger, 1977) AD A029 227
- PP 160  
Wilson, Desmond P., Jr., "The U.S. Sixth Fleet and the Conventional Defense of Europe," 50 pp., Sep 1976 (Submitted for publication in Adelphi Papers, I.I.S.S., London) AD A030 457
- PP 161  
Melich, Michael E. and Peet, Vice Adm. Ray (USN, Retired), "Fleet Commanders: Afloat or Ashore?" 9 pp., Aug 1976 (Reprinted from U.S. Naval Institute Proceedings, Jun 1976) AD A030 456
- PP 162  
Friedheim, Robert L., "Parliamentary Diplomacy," 106 pp. Sep 1976 AD A033 306
- PP 163  
Lockman, Robert F., "A Model for Predicting Recruit Losses," 9 pp., Sep 1976 (Presented at the 84th annual convention of the American Psychological Association, Washington, D.C., 4 Sep 1976) AD A030 459
- PP 164  
Mahoney, Robert B., Jr., "An Assessment of Public and Elite Perceptions in France, The United Kingdom, and the Federal Republic of Germany, 31 pp., Feb 1977 (Presented at Conference "Perception of the U.S. - Soviet Balance and the Political Uses of Military Power" sponsored by Director, Advanced Research Projects Agency, April 1976) AD 036 599
- PP 165  
Jondrow, James M. "Effects of Trade Restrictions on Imports of Steel," 67 pp., November 1976, (Delivered at ILAB Conference in Dec 1976)
- PP 166  
Feldman, Paul, "Impediments to the Implementation of Desirable Changes in the Regulation of Urban Public Transportation," 12 pp., Oct 1976, AD A033 322
- PP 166 - Revised  
Feldman, Paul, "Why It's Difficult to Change Regulation," Oct 1976
- PP 167  
Kleinman, Samuel, "ROTC Service Commitments: a Comment," 4 pp., Nov 1976, (To be published in Public Choice, Vol. XXIV, Fall 1976) AD A033 305
- PP 168  
Lockman, Robert F., "Revalidation of CNA Support Personnel Selection Measures," 36 pp., Nov 1976
- PP 169  
Jacobson, Louis S., "Earnings Losses of Workers Displaced from Manufacturing Industries," 38 pp., Nov 1976, (Delivered at ILAB Conference in Dec 1976), AD A039 808
- PP 170  
Brechling, Frank P., "A Time Series Analysis of Labor Turnover," Nov 1976, (Delivered at ILAB Conference in Dec 1976)
- PP 171  
Ralston, James M., "A Diffusion Model for GaP Red LED Degradation," 10 pp., Nov 1976, (Published in Journal of Applied Physics, Vol. 47, pp. 4518-4527, Oct 1976)

- PP 172**  
**Classen, Kathleen P.**, "Unemployment Insurance and the Length of Unemployment," Dec 1976, (Presented at the University of Rochester Labor Workshop on 16 Nov 1976)
- PP 173**  
**Kleinman, Samuel D.**, "A Note on Racial Differences in the Added-Worker/Discouraged-Worker Controversy," 2 pp., Dec 1976, (Published in the American Economist, Vol. XX, No. 1, Spring 1976)
- PP 174**  
**Mahoney, Robert B., Jr.**, "A Comparison of the Brookings and International Incidents Projects," 12 pp., Feb 1977 AD 037 206
- PP 175**  
**Levine, Daniel; Stoloff, Peter and Spruill, Nancy**, "Public Drug Treatment and Addict Crime," June 1976, (Published in Journal of Legal Studies, Vol. 5, No. 2)
- PP 176**  
**Felix, Wendi**, "Correlates of Retention and Promotion for USNA Graduates," 38 pp., Mar 1977, AD A039 040
- PP 177**  
**Lockman, Robert F. and Warner, John T.**, "Predicting Attrition: A Test of Alternative Approaches," 33 pp., Mar 1977, (Presented at the OSD/ONR Conference on Enlisted Attrition Xerox International Training Center, Leesburg, Virginia, 4-7 April 1977), AD A039 047
- PP 178**  
**Kleinman, Samuel D.**, "An Evaluation of Navy Unrestricted Line Officer Accession Programs," 23 pp., April 1977, (To be presented at the NATO Conference on Manpower Planning and Organization Design, Stressa, Italy, 20 June 1977), AD A039 048
- PP 179**  
**Stoloff, Peter H. and Balut, Stephen J.**, "Vacate: A Model for Personnel Inventory Planning Under Changing Management Policy," 14 pp., April 1977, (Presented at the NATO Conference on Manpower Planning and Organization Design, Stressa, Italy, 20 June 1977), AD A039 049
- PP 180**  
**Horowitz, Stanley A. and Sherman, Allan**, "The Characteristics of Naval Personnel and Personnel Performance," 16 pp., April 1977, (Presented at the NATO Conference on Manpower Planning and Organization Design, Stressa, Italy, 20 June 1977), AD A039 050
- PP 181**  
**Balut, Stephen J. and Stoloff, Peter**, "An Inventory Planning Model for Navy Enlisted Personnel," 35 pp., May 1977, (Prepared for presentation at the Joint National Meeting of the Operations Research Society of America and The Institute for Management Science, 9 May 1977, San Francisco, California), AD A042 221
- PP 182**  
**Murray, Russell, 2nd**, "The Quest for the Perfect Study or My First 1138 Days at CNA," 57 pp., April 1977
- PP 183**  
**Kassing, David**, "Changes in Soviet Naval Forces," 33 pp., November, 1976, (Published as part of Chapter 3, "General Purpose Forces: Navy and Marine Corps," in Arms, Men, and Military Budgets, Francis P. Hoeber and William Schneider, Jr. (eds.), (Crane, Russak & Company, Inc.: New York, 1977)), AD A040 106
- PP 184**  
**Lockman, Robert F.**, "An Overview of the OSD/ONR Conference on First Term Enlisted Attrition," 22 pp., June 1977, (Presented to the 39th MORS Working Group on Manpower and Personnel Planning, Annapolis, Md., 28-30 June 1977), AD A043 618
- PP 185**  
**Kassing, David**, "New Technology and Naval Forces in the South Atlantic," 22 pp., (This paper was the basis for a presentation made at the Institute for Foreign Policy Analyses, Cambridge, Mass., 28 April 1977), AD A043 619
- PP 186**  
**Mizrahi, Maurice M.**, "Phase Space Integrals, Without Limiting Procedure," 31 pp., May 1977, (Invited paper presented at the 1977 NATO Institute on Path Integrals and Their Application in Quantum Statistical, and Solid State Physics, Antwerp, Belgium, July 17-30, 1977) (Published in Journal of Mathematical Physics 19(1), p. 298, Jan 1978), AD A040 107
- PP 187**  
**Coile, Russell C.**, "Nomography for Operations Research," 35 pp., April 1977, (Presented at the Joint National Meeting of the Operations Research Society of America and The Institute for Management Services, San Francisco, California, 9 May 1977), AD A043 620
- PP 188**  
**Durch, William J.**, "Information Processing and Outcome Forecasting for Multilateral Negotiations: Testing One Approach," 53 pp., May 1977 (Prepared for presentation to the 18th Annual Convention of the International Studies Association, Chase-Park Plaza Hotel, St. Louis, Missouri, March 16-20, 1977), AD A042 222
- PP 189**  
**Coile, Russell C.**, "Error Detection in Computerized Information Retrieval Data Bases," July 1977, 13 pp., Presented at the Sixth Cranfield International Conference on Mechanized Information Storage and Retrieval Systems, Cranfield Institute of Technology, Cranfield, Bedford, England, 26-29 July 1977, AD A043 580
- PP 190**  
**Mahoney, Robert B., Jr.**, "European Perceptions and East-West Competition," 96 pp., July 1977 (Prepared for presentation at the annual meeting of the International Studies Association, St. Louis, Mo., March, 1977), AD A043 661
- PP 191**  
**Sawyer, Ronald**, "The Independent Field Assignment: One Man's View," August 1977, 25 pp.
- PP 192**  
**Holen, Arlene**, "Effects of Unemployment Insurance Entitlement on Duration and Job Search Outcome," August 1977, 6 pp., (Reprinted from Industrial and Labor Relations Review, Vol. 30, No. 4, Jul 1977)
- PP 193**  
**Horowitz, Stanley A.**, "A Model of Unemployment Insurance and the Work Test," August 1977, 7 pp., (Reprinted from Industrial and Labor Relations Review, Vol. 30, No. 40, Jul 1977)
- PP 194**  
**Classen, Kathleen P.**, "The Effects of Unemployment Insurance on the Duration of Unemployment and Subsequent Earnings," August 1977, 7 pp., (Reprinted from Industrial and Labor Relations Review, Vol. 30, No. 40, Jul 1977)
- PP 195**  
**Brechling, Frank**, "Unemployment Insurance Taxes and Labor Turnover: Summary of Theoretical Findings," 12 pp., (Reprinted from Industrial and Labor Relations Review, Vol. 30, No. 40, Jul 1977)
- PP 196**  
**Ralston, J. M. and Lorimer, O. G.**, "Degradation of Bulk Electroluminescent Efficiency in Zn, O-Doped GaP LEDs," July 1977, 3 pp., (Reprinted from IEEE Transactions on Electron Devices, Vol. ED-24, No. 7, July 1977)
- PP 197**  
**Wall, Anthony R.**, "The Centre for Naval Analyses," 14 pp., Dec 1977, AD A049 107
- PP 198**  
**Classen, Kathleen P.**, "The Distributional Effects of Unemployment Insurance," 26 pp., Sept. 1977 (Presented at a Hoover Institution Conference on Income Distribution, Oct 7-8, 1977)
- PP 199**  
**Durch, William J.**, "Revolution From A F.A.R. - The Cuban Armed Forces in Africa and the Middle East," Sep 1977, 16 pp., AD A046 268
- PP 200**  
**Powers, Bruce F.**, "The United States Navy," 40 pp., Dec 1977, (To be published as a chapter in The U.S. War Machine by Salamander Books in England during 1978), AD A049 108
- PP 201**  
**Durch, William J.**, "The Cuban Military in Africa and The Middle East: From Algeria to Angola," Sep 1977, 67 pp., AD A045 675
- PP 202**  
**Feldman, Paul**, "Why Regulation Doesn't Work," (Reprinted from Technological Change and Welfare in the Regulated Industries and Review of Social Economy, Vol. XXIX, March, 1971, No. 1, Sep 1977, 8 pp.)
- PP 203**  
**Feldman, Paul**, "Efficiency, Distribution, and the Role of Government in a Market Economy," (Reprinted from The Journal of Political Economy, Vol. 79, No. 3, May/June 1971, Sep 1977, 19 pp.), AD A045 675

- PP 204  
Wells, Anthony R., "The 1967 June War: Soviet Naval Diplomacy and The Sixth Fleet - A Re-appraisal," Oct 1977, 36 pp., AD A047 236
- PP 205  
Coile, Russell C., "A Bibliometric Examination of the Square Root Theory of Scientific Publication Productivity," (Presented at the annual meeting of the American Society for Information Science, Chicago, Illinois, 29 September 1977), Oct 1977, 6 pp., AD A047 237
- PP 206  
McConnell, James M., "Strategy and Missions of the Soviet Navy in the Year 2000," 48 pp., Nov 1977, (Presented at a Conference on Problems of Sea Power as we Approach the 21st Century, sponsored by the American Enterprise Institute for Public Policy Research, 6 October 1977, and subsequently published in a collection of papers by the Institute), AD A047 244
- PP 207  
Goldberg, Lawrence, "Cost-Effectiveness of Potential Federal Policies Affecting Research & Development Expenditures in the Auto, Steel and Food Industries," 36 pp., Oct 1977, (Presented at Southern Economic Association Meetings beginning 2 November 1977)
- PP 208  
Roberts, Stephen S., "The Decline of the Overseas Station Fleets: The United States Asiatic Fleet and the Shanghai Crisis, 1932," 18 pp., Nov 1977, (Reprinted from The American Neptune, Vol. XXXVII, No. 3, July 1977), AD A047 245
- PP 209 - Classified.
- PP 210  
Kassing, David, "Protecting The Fleet," 40 pp., Dec 1977 (Prepared for the American Enterprise Institute Conference on Problems of Sea Power as We Approach the 21st Century, October 6-7, 1977), AD A049 109
- PP 211  
Mizrahi, Maurice M., "On Approximating the Circular Coverage Function," 14 pp., Feb 1978
- PP 212  
Mangel, Marc, "On Singular Characteristic Initial Value Problems with Unique Solutions," 20 pp., Jun 1978 (To be submitted for publication in Journal of Mathematical Analysis and Its Applications)
- PP 213  
Mangel, Marc, "Fluctuations in Systems with Multiple Steady States. Application to Lanchester Equations," 12 pp., Feb 78, (Presented at the First Annual Workshop on the Information Linkage Between Applied Mathematics and Industry, Naval PG School, Feb 23-25, 1978)
- PP 214  
Weinland, Robert G., "A Somewhat Different View of The Optimal Naval Posture," 37 pp., Jun 1978 (Presented at the 1978 Convention of the American Political Science Association (APSA/IUS Panel on "Changing Strategic Requirements and Military Posture"), Chicago, Ill., September 2, 1978)
- PP 215  
Coile, Russell C., "Comments on: Principles of Information Retrieval by Manfred Kochen, 10 pp., Mar 78, (Published as a Letter to the Editor, Journal of Documentation, Vol. 31, No. 4, pages 298-301, December 1975)
- PP 216  
Coile, Russell C., "Lotka's Frequency Distribution of Scientific Productivity," 18 pp., Feb 1978, (Published in the Journal of the American Society for Information Science, Vol. 28, No. 6, pp. 368-370, November 1977)
- PP 217  
Coile, Russell C., "Bibliometric Studies of Scientific Productivity," 17 pp., Mar 78, (Presented at the Annual meeting of the American Society for Information Science held in San Francisco, California, October 1976.)
- PP 218 - Classified.
- PP 219  
Huntzinger, R. LaVar, "Market Analysis with Rational Expectations: Theory and Estimation," 60 pp., Apr 78 (To be submitted for publication in Journal of Econometrics)
- PP 220  
Maurer, Donald E., "Diagonalization by Group Matrices," 26 pp., Apr 78
- PP 221  
Weinland, Robert G., "Superpower Naval Diplomacy in the October 1973 Arab-Israeli War," 76 pp., Jun 1978
- PP 222  
Mizrahi, Maurice M., "Correspondence Rules and Path Integrals," 30 pp., Jun 1978 (Invited paper presented at the CNRS meeting on "Mathematical Problems in Feynman's Path Integrals," Marseille, France, May 22-26, 1978)
- PP 223  
Mangel, Marc, "Stochastic Mechanics of Molecule-Molecule Reactions," 21 pp., Jun 1978 (To be submitted for publication in Journal of Mathematical Physics)
- PP 224  
Mangel, Marc, "Aggregation, Bifurcation, and Extinction In Exploited Animal Populations," 48 pp., Mar 1978 (To be submitted for publication in American Naturalist)  
"Portions of this work were started at the Institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, B.C., Canada"
- PP 225  
Mangel, Marc, "Oscillations, Fluctuations, and the Hopf Bifurcation," 43 pp., Jun 1978  
"Portions of this work were completed at the Institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, Canada."
- PP 226  
Ralston, J. M. and J. W. Mann\*, "Temperature and Current Dependence of Degradation in Red-Emitting GaP LEDs," 34 pp., Jun 1978
- PP 227  
Mangel, Marc, "Uniform Treatment of Fluctuations at Critical Points," 50 pp., May 1978 (To be submitted for publication in Journal of Statistical Physics)
- PP 228  
Mangel, Marc, "Relaxation at Critical Points: Deterministic and Stochastic Theory," 54 pp., Jun 1978 (To be submitted for publication in Journal of Mathematical Physics)
- PP 229  
Mangel, Marc, "Diffusion Theory of Reaction Rates, I: Formulation and Einstein-Smoluchowski Approximation," 50 pp., Jan 1978
- PP 230  
Mangel, Marc, "Diffusion Theory of Reaction Rates, II Ornstein-Uhlenbeck Approximation," 34 pp., Feb 1978
- PP 231  
Wilson, Desmond P., Jr., "Naval Projection Forces: The Case for a Responsive MAF," Aug 1978
- PP 232  
Jacobson, Louis, "Can Policy Changes be Made Acceptable to Labor?" Aug 1978 (To be submitted for publication in Industrial and Labor Relations Review)
- PP 233  
Jacobson, Louis, "An Alternative Explanation of the Cyclical Pattern of Quits," 23 pp., Sep 1978
- PP 234  
Jondrow, James and Levy, Robert A., "Does Federal Expenditure Displace State and Local Expenditure: The Case of Construction Grants," 18 pp., Oct 1978 (To be submitted for publication in Journal of Public Economics)
- PP 235  
Mizrahi, Maurice M., "The Semiclassical Expansion of the Anharmonic-Oscillator Propagator," 41 pp., Oct 1978 (To appear in the Journal of Mathematical Physics)
- PP 237  
Maurer, Donald, "A Matrix Criterion for Normal Integral Bases," 10 pp., Jan 1979
- PP 238  
Ungoff, Kathleen Classen, "Unemployment Insurance and The Employment Rate," 20 pp., Oct 1978
- PP 240  
Powers, Bruce, "Goals of the Center for Naval Analyses," 13 pp., December 1978
- PP 241  
Mangel, Marc, "Fluctuations at Chemical Instabilities," 24 pp., Dec 1978 (Published in Journal of Chemical Physics, Vol. 69, No. 8, Oct 15, 1978)
- PP 242  
Simpson, William R., "The Analysis of Dynamically Interactive Systems (Air Combat by the Numbers)," 160 pp., Dec 1978

PP 243

Simpson, William R., "A Probabilistic Formulation of Murphy Dynamics as Applied to the Analysis of Operational Research Problems, 18 pp., Dec 1978 (Submitted for publication in The Journal of Irreproducible Results)

PP 244

Sherman, Allan, Horowitz, Stanley A., "Maintenance Costs of Complex Equipment," 20 pp., Dec 1978

PP 248

Thomas, James A., Jr., "The Transport Properties of Dilute Gases in Applied Fields," 183 pp., Mar 1979

PP 249

Glasser, Kenneth S., "A Secretary Problem with a Random Number of Choices," 23 pp., Mar 1979 (Submitted for publication in Journal of the American Statistical Association)